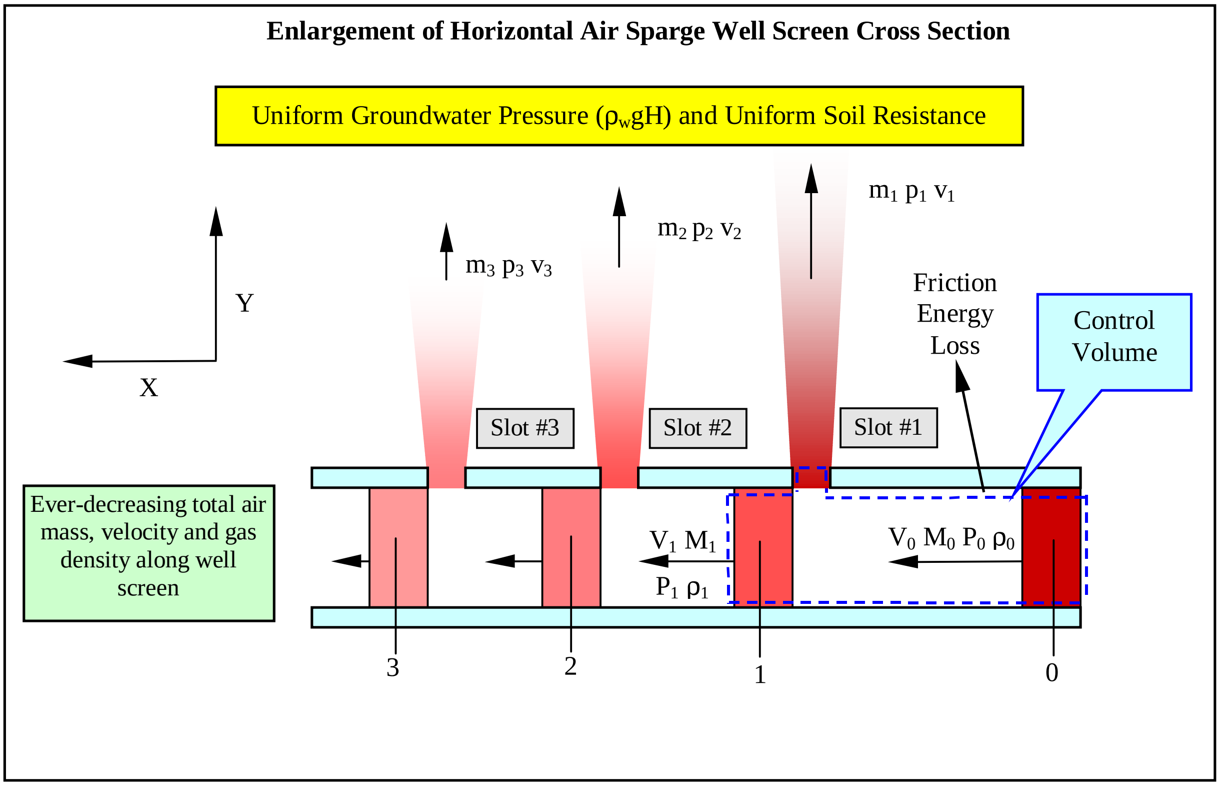
PART 2

III. Application of Newtonian Physics to Distributed Compressible Fluid Flow in Horizontal Wells

If one takes *Bernoulli's Equation* and applies it to a horizontally placed well in which *air* is being sparged into uniform soil, the graphical model for *a short segment of well screen* could be illustrated by the following:



In this model and ensuing equations, several simplifications and assumptions have been made. Though the graphically represented simplifications remain valid in the "real world", the simplifications and simplifying assumptions made in the ensuing equations do not "carry over" with great accuracy in an actual installation. However, these mathematical assumptions and simplifications greatly reduce the complexity of the equations involved, and without such action the equations and ensuing discussion about them would take many pages of text and would be extremely involved. Suffice it to say that the aforementioned model and simplified equations can be used for illustrative purposes.

In this model, the *control volume* across the first slot in the well is shown. To simplify the model, it will be assumed that there are no slots in this or any other successive plane going into or out of the paper (e.g., the effect of the Z-axis is ignored). Further, a convention will be defined where positive Y values will progress up and positive X values will be from right to left.

First let's *assume* that some *known* static pressure $\mathbf{P_0}$ is applied to the well which results in some *unknown* total mass of air $\mathbf{M_0}$, at some *unknown* mass density $\boldsymbol{\rho_0}$ and some *unknown* velocity $\mathbf{V_0}$ entering the well from the right. **Slot #1** possesses some finite open area to the well's interior surface, and by applying *yet-developed* equations to account for the *assumed* uniform flow resistance of the soil and static pressure of ground water, some incremental mass $\mathbf{m_1}$ exits this slot. Upon exiting, mass $\mathbf{m_1}$ (which arguably possesses some static pressure $\mathbf{p_1}$ equal to that of static groundwater and occupies some finite volume) expands as it rises and disperses through to the surface of the water table. Since some finite mass "escapes" from the slot, *the total remaining mass* in the well downstream of this slot is reduced by mass $\mathbf{m_1}$. Mass $\mathbf{M_1}$ represents the remaining mass within the well. Further, as the "slug" of air progresses across the slot (and mass $\mathbf{m_1}$ escapes from the slot) friction between the air and the walls of the well occurs. This friction results in a net *loss* of energy *to* the surroundings, which manifests itself (ultimately) in heat loss from the air through the well screen wall to the soil.

Because some mass leaves the control volume (through the slot), the in-well mass remaining after the slot must be less than that before the slot (e.g., $\mathbf{\Sigma M_1 < \Sigma M_0}$). Further, since this exiting mass possesses some energy *and* friction results in additional energy loss to the surroundings, the resulting total energy level of the remaining mass $\mathbf{M_1}$ must be less than $\mathbf{M_0}$. Using similar nomenclature, $\mathbf{\Sigma E_1 < \Sigma E_0}$. Since this energy manifests itself in both *kinetic* and *potential energy* forms, the net reduction in total energy results in static pressure $\mathbf{P_1}$ and velocity $\mathbf{V_1}$ being lower than $\mathbf{P_0}$ and $\mathbf{V_0}$ respectively.

For illustrative purposes, if it is assumed that *Bernoulli's Equation* for *incompressible flow* (derived in the previous section) can be applied to this compressible flow example (to greatly simplify matters), the previous discussion of mass and energy across the control volume can be placed in equation form. Taking this liberty, the following equation can be formulated between points **0** and **1**:

$$M_0\left(\frac{P_0}{\gamma_0}+\frac{V_0^2}{2g}+Z_0\right)=M_1\left(\frac{P_1}{\gamma_1}+\frac{V_1^2}{2g}+Z_1\right)+m_1\sum Energy_1+HL_0^1$$

Since friction loss **HL** is a function of the well's internal surface roughness, inside diameter and the flow's *Reynold's Number*, per the *Darcy Weisbach* Equation:

$$HL_0^1 = \bar{f}_0^1 \frac{L_0^1}{D} \cdot \frac{\left(\frac{V_0 + V_1}{2}\right)^2}{2g}$$

In this equation **L** is the distance across the control volume, **D** is the well screen inside diameter and **f** is by definition a *friction factor* that is a function of the well interior's surface roughness, the *kinematic viscosity* of the air (at the conditions inside the control volume), well diameter, length of "element", and the average velocity across that element. The value of **f** is typically found by using a *Moody Diagram* (it can also be determined through a series of 3 iterative equations). Substituting, for the simple element of a screen segment consisting of 1 slot, the equation of state becomes:

$$M_0\left(\frac{P_0}{\gamma_0} + \frac{V_0^2}{2g} + Z_0\right) = M_1\left(\frac{P_1}{\gamma_1} + \frac{V_1^2}{2g} + Z_1\right) + m_1 \sum Energy_1 + \bar{f}_0^1 \frac{L_0^1}{D} \cdot \frac{\left(\frac{V_0 + V_1}{2}\right)^2}{2g}$$

Since the previous equation represents the mass and energy balance across but only *1 slot*, and there are many slots in the typical horizontal well, it follows that in order to determine the mass-energy balance for the entire well, the previous equation will have to be expanded to include all slots to the well's end.

By inspection, when one pursues this action, it becomes clear that the left side of the equation will remain as stated (since the system entrance conditions remain the same regardless of how many slots or how much of the well is analyzed). In addition, the terms for the mass leaving **Slot #1** and the energy loss (due to friction) between points **0** and **1** will remain as stated. What then needs to be added, or more precisely what needs to be *expanded,* is the **M_1(…)** expression. Recall that the **M_1(…)** expression represents the mass *and* energy of what remains *inside the well* after **Slot #1**. If the entire well is analyzed to its last (or **nth)** slot, there will be no mass and hence no associated energy remaining inside the well. All mass will have escaped through slots and all energy will have been expended through friction and the energy of each incremental (but not necessarily equal) mass exiting the well's slots. Therefore, the previous equation expanded to encompass the mass-energy balance *of the entire well* will become as follows:

$$M_0\left(\frac{P_0}{\gamma_0}+\frac{V_0^2}{2g}+Z_0\right)=\sum_{i=1}^{n}\left(m_i\sum Energy_i\right)+\sum_{i=1}^{n}\left[\bar{f}_{i-1}^{\,i}\,\frac{L_{i-1}^{i}}{D}\cdot\frac{\left(\frac{V_{i-1}+V_i}{2}\right)^2}{2g}\right]$$

If this equation is expanded a little further, to provide "quantification" of the **Energy $_i$** term into its constituent components, the mass-energy equation for the well becomes:

$$M_0\left(\frac{P_0}{\gamma_0}+\frac{V_0^2}{2g}+Z_0\right)=\sum_{i=1}^{n}\left(m_i\sum\left(\frac{P_i}{\gamma_i}+\frac{v_i^2}{2g}+Z_i\right)\right)+\sum_{i=1}^{n}\left[\bar{f}_{i-1}^{\,i}\,\frac{L_{i-1}^{i}}{D}\cdot\frac{\left(\frac{V_{i-1}+V_i}{2}\right)^2}{2g}\right]$$

For this final equation a new term $\mathbf{v_i}$ is introduced. This term differs from $\mathbf{V_i}$ in that $\mathbf{v_i}$ is, by definition, the velocity of each incremental $\mathbf{m_i}$ mass as it leaves each slot. $\mathbf{V_i}$, by comparison, is the velocity of the remaining gas (e.g., $\mathbf{M_i}$) *inside the well after* **Slot$_i$**, which is <u>much different</u> in value from $\mathbf{v_i}$.

In attempting to solve this equation several issues present themselves:

1. If our well screen was, say 200 ft. long and was fabricated with 4 rows of slots with 25 slots per row per foot of screen the value of **n** in the equation would be 20,000 (e.g., 4 x 25 x 200 = 20,000). There would <u>have</u> to be 20,000 separate and unique $\mathbf{m_i}$, $\mathbf{P_i}$, $\boldsymbol{\gamma_i}$, $\mathbf{v_i}$, $\mathbf{V_i}$, $\mathbf{Z_i}$, and **f** terms, resulting in the equation (when expanded to encompass all these terms) being *huge* by anyone's standards!
   - Each value of $\mathbf{m_i}$ and $\mathbf{v_i}$ is <u>unknown</u> since we have yet to develop the equation(s) to calculate the mass *that can be discharged* (per slot) into the soil.
   - The friction factor **f** which, in part, is based on the in-well flow *velocity* and *density* is <u>different</u> for each increment (e.g., for each value of **i**) since both the *velocity* <u>and</u> *density* of the in-well gas changes each time mass exits the well (through any slot). In fact (and contrary to common belief), the friction factor <u>increases</u> rather than decreases with distance along the well screen.
   - Each $\mathbf{P_i}$ and $\mathbf{V_i}$ term is different for each value of **i** and are all <u>unknown</u> at this point.

- Further, each incremental weight density γ_i is unknown, since each incremental pressure P_i is unknown.
- But, if the well screen is perfectly horizontal as installed in the ground each Z_i term becomes a constant.

2. If the distance between slots is uniform the value of **L** (in the friction energy expression) becomes a constant, which aids in solving the equation.
3. If the well's inside diameter is constant over the length of the screen the value of **D** (also in the friction energy expression) becomes a constant as well, thus aiding in our attempts to solve the equation.
4. The total mass input into the system M_0 is unknown but it is known that M_0 must equal the sum of all m_i masses.
5. The inlet velocity of the system V_0 is unknown, since it is not known how much mass M_0 enters the system. Further, it is known that the in-well velocity at the **nth** slot (e.g., V_n) must be zero.
6. Finally, the input pressure P_0, though not yet quantified can be specified as a specific numeric value.

Where these issues leave us is with the realization that our developed mass-energy balance equation for a simplified horizontal well model is very complicated. In this singular equation there are minimally (and realistically) many thousand variables whose values we yet do not know and which have to be determined to solve the equation. Further, several of these variables are "related", such that some are functions of others (such as weight density γ_i is a function of each incremental pressure P_i). Finally, with all these difficulties, we still cannot determine the incremental mass discharged from each slot m_i, which is realistically what the goal should be in this exercise (e.g., to determine how much air is injected into the soil from our well)! In deriving the mass-energy balance equation for our system the presumption was that we *knew* the value of each m_i or could "easily" find it. In fact, we *do not* know any of the m_i values and don't even know what the initial M_0 quantity is either!

To determine the value of each m_i we would have to derive *additional* equations that quantify how much mass can escape from *each* slot (of a chosen geometrical configuration) taking into consideration that this mass is *resisted from escaping* the well due to the inherent resistance of the soil and the static pressure of groundwater. Thereafter, if we *could* determine all the m_i values, we could then sum them to determine M_0. Thereafter we can *verify* the initial entrance velocity V_0 and determine if the velocity at the final slot V_n is zero. We could then check all of the M_i values to see if the friction loss along the well screen and resultant P_i terms match those we used to calculate each m_i. If so, our system is balanced thermodynamically (e.g., by mass and energy) and in accordance with the Laws of Newtonian Physics, and we would know *exactly* how our horizontal well injection system will perform. If not, then along the way either analytical errors were made, incorrect simplifying assumptions were applied, and/or too many assumptions invalid for the conditions at hand were made. In each of these cases the analysis must begin anew as a new iteration and must be carried out to conclusion and repeated as necessary until the system <u>exactly</u> balances.